Controlling chaos in electronic circuits

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Chaotic oscillations in electronic circuits are rather an unwanted phenomenon. We describe various control concepts the goal of which is to suppress chaos and to achieve a desired type of dynamic behaviour such as stable fixed point or a periodic orbit. The control concepts described here include: (i) system parameter variation; (ii) chaotic oscillation absorber; (iii) entrainment; (iv) linear feedback control; (v) time-delayed feedback, (vi) methods for stabilizing unstable periodic orbits: occasional proportional feedback and sampled input waveform methods. Advantages and drawbacks of these methods are described. Control towards chaotic states having several potential applications is also considered.

1. Introduction

Research efforts during the past decade have lead to a general understanding of a variety of nonlinear phenomena in electronic circuits. In particular many unwanted phenomena as excess noise, false frequency lockings, squegging, phase slipping have been found to be associated with bifurcations and chaotic behaviour. Also many nonlinear phenomena in other science and engineering disciplines have a strong link with 'electronic chaos'; let us mention here the most spectacular ones: heart fibrillation (electrocardiogram patterns) and epileptic waveforms in electroencephalographic patterns. Chaos, so commonly encountered in electronic circuits and systems, represents rather a peculiar type of behaviour commonly considered as causing malfunctioning, disastrous and thus unwanted in most applications. It is obvious that an amplifier, a filter, an A/D converter, a phase-locked loop or a digital filter generating chaotic responses is of no use, at least for its original purpose. Similarly we would like to avoid situations where the heart does not pump blood properly (fibrillation or arhythmias) or epileptic attacks. Thus a most common goal of control for chaotic electronic circuits is suppression of oscillations of the 'bad' kind and influencing it in such a way that it will produce a prescribed, desired motion. The goals vary depending on a particular application. The most common goal is to convert chaotic motion into a stable periodic or constant one. In some cases elimination of multiple basins of attraction is desirable. Yet another goal could be tracking (synchronization) to another kind of chaotic trajectory.

Considering the possibilities of influencing the dynamics of a chaotic circuit one can distinguish four basic approaches: (i) variation of an existing accessible system parameter; (ii) change in the system design, modification of its internal structure; (iii) injection of an external signal(s); (iv) introduction of a controller (classical PI, PID, linear or nonlinear, neural, stochastic, etc.).

Due to very rich dynamic phenomena encountered in typical chaotic systems, there exist a large variety of approaches to controlling such systems. This paper presents selected methods developed for controlling chaos in various aspects, starting from the most primitive concepts like parameter variation, through classical controller applications (open- and closed-loop control), to quite sophisticated ones like stabilization of unstable periodic orbits embedded within the strange attractor.

2. Simple techniques for suppressing chaotic oscillations: change in the system design

(a) *Effects of large parameter changes*

The simplest way of suppressing chaotic oscillations is to change the system parameters (redesign!) in such a way as to produce the desired kind of behaviour. The influence of parameter variations on the asymptotic behaviour of the system can be studied using a standard tool used in analysis of chaotic systems, the bifurcation diagram. Typical bifurcation diagram reveals a variety of dynamic behaviours for appropriate choices of system parameters and tells us what parameter values should be chosen to obtain the desired goal behaviour. In electronic circuits changes in the dynamic behaviour are obtained by changing one of its passive elements values (which means replacing one of the resistors, capacitors, or in rare cases inductances). This method, although intuitively simple has a major drawback: it requires large parameter variations ('large energy control'). This requirement cannot be met in many physical systems where the construction parameters are either fixed or can be changed in very small ranges. This method is also difficult to apply on the design stage as there are no simulation tools for electronic circuits allowing bifurcation analysis (e.g. SPICE has no such capability). On the other hand programs offering such types of analysis require a description of the problem in closed mathematical form as differential or difference equations. Changes of parameters are even more difficult to introduce once the circuitry is fabricated or breadboarded and if possible at all can be done on trial-and-error basis.

(b) *'Shock absorber' concept: change in system structure*

This simple technique is being used in a variety of applications. The motivation comes from mechanical engineering where devices absorbing unwanted vibrations are commonly used (e.g. beds of machine-tools, shock-absorbers in vehicle suspensions, etc.). The idea is to modify the original chaotic system design (add the 'absorber' without major changes in the design or construction) in order to change its dynamics in such a way that a new stable orbit appears in a neighbourhood of the original chaotic attractor. In an electronic system the absorber can be as simple as an additional shunt capacitor or a LC tank circuit. Kapitaniak *et al.* (1992) proposed such a 'chaotic oscillation absorber' for Chua's circuit; it is a parallel RLC circuit coupled with original Chua's circuit via the resistor, depending on its value the original chaotic behaviour can be converted to a chosen stable oscillation.

3. External perturbation techniques

Several authors have demonstrated that a chaotic system can be forced to perform in a desired way by injecting external signals that are independent on the internal variables or structure of the system. Three types of such signal were considered: (*a*) aperiodic signals ('resonant stimulation'); (*b*) periodic signals of small amplitude; (*c*) external noise.

(*a*) *'Entrainment': open loop control*

Aperiodic external driving was one of the first methods introduced by Hübler (1988) (termed 'resonant stimulation') and developed in the works of Hübler & Jackson (see references in Jackson 1991). A mathematical model of the considered experimental system is needed (e.g. in the form of differential equation: $\mathrm{d}x/\mathrm{d}t = F(x)$, $x \in R^n$, $F(x)$; differentiable, solutions exist for every $t > 0$).

The goal of the control is to entrain the solution $x(t)$ to an arbitrarily chosen behaviour $g(t)$, i.e.

$$\lim_{t\to\infty} |x(t) - g(t)| = 0. \tag{3.1}$$

Entrainment can be obtained by injecting the control signal:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = F(x) + (\dot{g} - F(g))1(t). \tag{3.2}$$

The entrainment method has the great advantage that no feedback is required and no parameters are being changed; thus the control signal can be computed in advance and no equipment for measurements of the state of the system are needed. The goal does not depend on the considered system and in fact it could be any signal at all (except solutions of the autonomous system – for $\dot{g} - F(g)$ – no control). It should be noticed, however, that this method has limited application since a good model of the system dynamics is necessary, and the set of initial states for which the system trajectories will be entrained is not known.

(*b*) *Weak periodic perturbation*

Interesting results have been reported by Breiman & Goldhirsch (1991), who studied effects of adding a small periodic driving signal in a system behaving in a chaotic way. They discovered that external sinusoidal perturbation of small amplitude and appropriately chosen frequency can eliminate chaotic oscillations in a model of the dynamics of a Josephson junction and cause the system to operate in some stable periodic mode. Under Hübler's direction Kennedy and Kozek built in Chua's laboratory in 1992 a control system using weak periodic perturbation to stabilize flow in a dripping faucet. Unfortunately there is little theory behind this approach and the goal behaviours can be learned by trial-and-error only. Some hope for further understanding and applications can be based on using theoretical results known from the theory of synchronization.

(*c*) *Noise injection*

Noise signal of small amplitude injected in a suitable way into the circuit (system) offers potentially new possibilities for stabilization of chaos. First observations date back to the work of Herzel (1988) and effects of noise injection were also studied in an RC-ladder chaotic oscillator in Ogorzałek & Mosekilde (1989). This approach although promising needs further investigations as little theory is available to support experimental observations.

4. Control engineering approaches

Sevaral attempts have been made to use known methods belonging to the 'control engineer's toolkit'. For example the use of PI and PID controllers for chaotic circuits, applications of stochastic control techniques, Lyapunov-type methods, robust controllers and many other methodologies including intelligent control and neural controllers have been described in literature (the paper by Chen & Dong (1993) and ch. 5 in Madan (1993) give an excellent review of applications of such methods). In electronic circuits two schemes: linear feedback and time-delay feedback seem to find most successful applications.

(a) *Error-feedback control*

Several methods of chaos control have been developed which relay on a common principle that the control signal is some function of the difference between the actual system output $x(t)$ and the desired goal dynamics $y(t)$. This control signal could be an actual system parameter: $p(t) = \phi(x(t) - y(t))$ as proposed in the 'adaptive control scheme' described by Huberman & Lumer (1990), or additive signal produced by a linear controller $u(t) = K(x(t) - y(t))$ as in the methodology developed by Pyragas (1992) and Chen & Dong (1993). Using error feedback chaotic motion has been successfully converted into periodic one both in discrete- and continuous-time systems. In particular chaotic motions in Duffing's oscillator and Chua's circuit have been controlled/directed towards fixed points or periodic orbits (Chen & Dong 1993). The important properties of the linear feedback chaos control method are: very simple structure of controller, access to system parameters is not required. The method is immune to small parameter variations but might be difficult to apply in real systems (interactions of many system variables needed). The choice of the goal orbit poses the most important problem; usually the goal is chosen in multiple experiments or can be specified on the basis of model calculations.

(b) *Time-delay feedback control (Pyragas method)*

Pyragas (1992) proposed an interesting feedback structure using a delayed copy of the output signal. He obtained very promising results in the control of many different chaotic systems. Despite the lack of mathematical rigour, this method is being successfully used in several applications. An interesting application of this technique is described by Mayer-Kress *et al.* (1993); Pyragas's control scheme has been used for tuning chaotic Chua's circuits to generate musical tones and signals. More recently Celka (1994) used Pyragas's method to control a real optical system.

The positive features of the delay feedback control method are: no external signals injected and no access to system parameters is required. Action of the control is immune to small parameter variations. In real electronic systems a variable delay element required is readily available (analogue delay lines are available as of-the-shelf components). Among drawbacks let us mention that there is no *a priori* knowledge of the goal (goal chosen by trial-and-error).

5. Control in terms of stabilizing unstable periodic orbits

(a) *Ott–Grebogi–Yorke approach (OGY)*

Ott *et al.* (1990) proposed a feedback method aiming at stabilization of a chosen unstable periodic orbit existing within the attractor (existence of a dense set of unstable periodic orbits is one of the key properties of chaotic attractors). To visualize best how the method works let us assume that the dynamics of the system are described by a k-dimensional map: $\xi_{n+1} = F(\xi_n, p)$, $\xi_i \in R^k$. This map in the case of continuous-time systems can be constructed, for example, by introducing a transversal surface of section for system trajectories, p is some accessible system parameter which can be changed in some small neighbourhood of its nominal value p^*. To explain the method we will concentrate now on stabilization of period-one orbit. Let $\xi_0 = F(\xi_0, p^*)$ be the chosen fixed point (period-1) of the map around which we would like to stabilize the system. Assume further that the position of this orbit changes smoothly with p parameter changes (i.e. p^* is not a bifurcation value), and there are little changes in the local system behaviour for small variations of p. In a close vicinity of this fixed point with good accuracy we can assume that the dynamics is linear and can be expressed approximately by

$$\xi_{n+1} - \xi_0 = A(\xi_n - \xi_0) + g(p_n - p^*). \tag{5.1}$$

The elements of the matrix

$$A = \frac{\partial F}{\partial \xi}(\xi_0, p^*)$$

and vector

$$g = \frac{\partial F}{\partial p}(\xi_0, p^*)$$

can be calculated by using the measured chaotic time series and analysing its behaviour in the neighbourhood of the fixed point. Further the eigenvalues λ_s, λ_u and eigenvectors e_s, e_u of this matrix can be found. These eigenvectors determine the stable and unstable directions in the small neighbourhood of the fixed point. Let us denote by f_s, f_u the contravariant eigenvectors ($f_s^{\mathrm{T}} e_s = f_u^{\mathrm{T}} e_u = 1$, $f_s^{\mathrm{T}} e_u = f_u^{\mathrm{T}} e_s = 0$). The control idea now is to monitor the system behaviour until it comes close to the desired fixed point (we assume that the system is ergodic and the trajectory fills densely the attractor; thus eventually it will pass arbitrarily close to any chosen point) and then change p by a small amount so the next state ξ_{n+1} should fall on the stable manifold of ξ_0, i.e. choose p_n such that $f_u^{\mathrm{T}}(\xi_{n+1} - \xi_0) = 0$:

$$p_n = -\left(\frac{\lambda_u}{f_u^{\mathrm{T}} g}\right) f_u^{\mathrm{T}}(\xi_n - \xi_0) + p^*. \tag{5.2}$$

Figure 1 schematically explains the action of the OGY algorithm in the case $k = 2$. The OGY technique has the notable advantage of not requiring analytical models of the system dynamics and is well-suited for experimental systems. One can use either full information from the process or use delay coordinate embedding technique using single variable experimental time series (see Dressler & Nitsche 1992). The procedure can also be extended to higher-period orbits. Any accessible variable (controllable) system parameter can be used for applying perturbation and the control signals are very small.

We have carried out an extensive study of application of the OGY technique to controlling chaos in Chua's circuit. Using an application-specific software package (Dąbrowski *et al.* 1994) we were able to find some of the unstable periodic orbits embedded in the double scroll chaotic attractor and use them further as control goals. Figure 2 shows the results of stabilization of a period-one and period-two unstable periodic orbits. Before control is achieved, the trajectories exhibit chaotic transients.

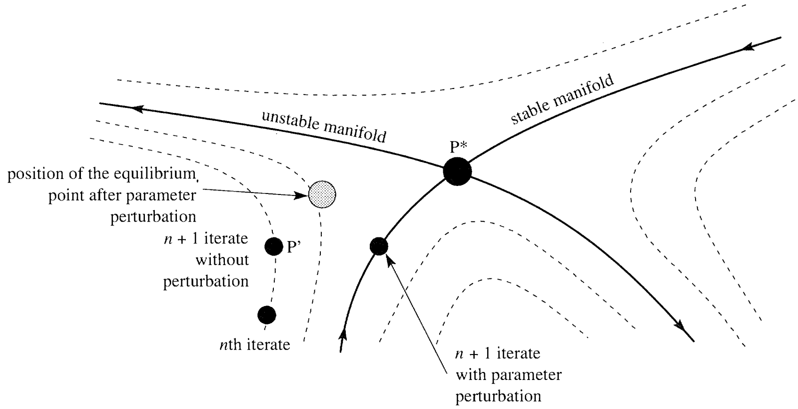

Figure 1. Schematic explanation of the OGY algorithm; a linear perturbation is applied in such a way that the successive iterate falls onto the stable manifold of the fixed point.

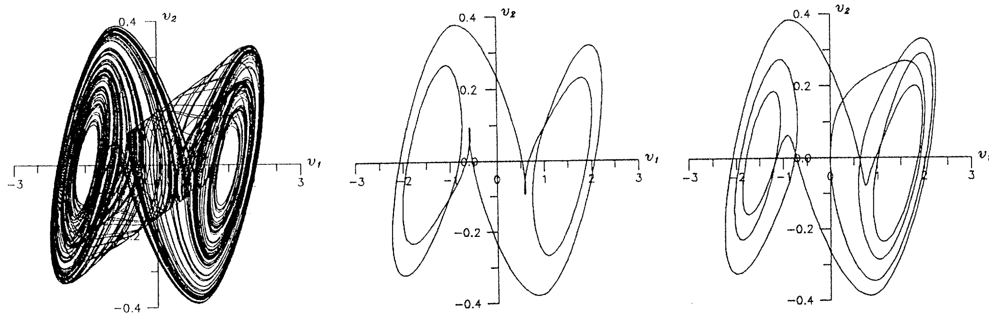

Figure 2. Double scroll chaotic attractor in Chua's circuit. Two of the abundance of periodic orbits that could be stabilized using OGY technique.

When applying the OGY method to control chaos in a real physical circuit the main problem was the error introduced by: inevitable noise of the circuit elements, A/D and D/A conversion of signals (quantification), rounding operations in the computer calculations, etc. The method was found to be very sensitive to the noise level; very small control signals sometimes are hidden within the noise and control is impossible.

(*b*) *Sampled input waveform method*

A very simple, robust and effective method of chaos control in terms of stabilization of an unstable periodic orbit has been proposed by Dedieu & Ogorzałek (1994). A sampled version of the output signal corresponding to a chosen unstable periodic trajectory, uncovered from a measured time series is applied in the chaotic system causing the system to follow this desired orbit. In real systems this sampled version of the unstable periodic orbit can be programmed into a programmable waveform generator and used as the forcing signal. The block diagram of this control scheme is shown in figure 3.

The sampled input control method is very attractive as the goal of the control can be specified using analysis of output time-series of the system, access to system parameters not required. Control is immune to parameter variations, noise, scaling and quantization – robust operation. Instead of a controller we need a generator to synthesize the goal signal. Signal sampling reduces the memory requirements for

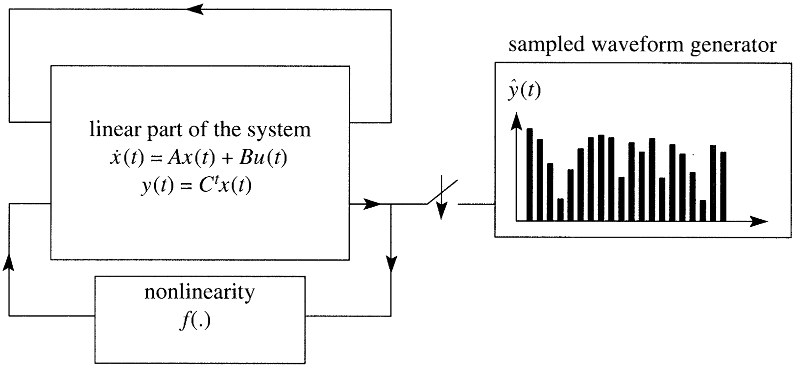

Figure 3. Block diagram of the sampled input chaos control system.

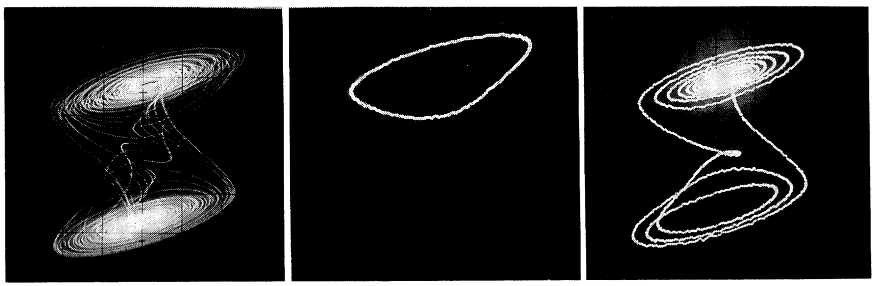

Figure 4. Double scroll chaotic attractor observed in the experimental system and two examples of controlled orbits.

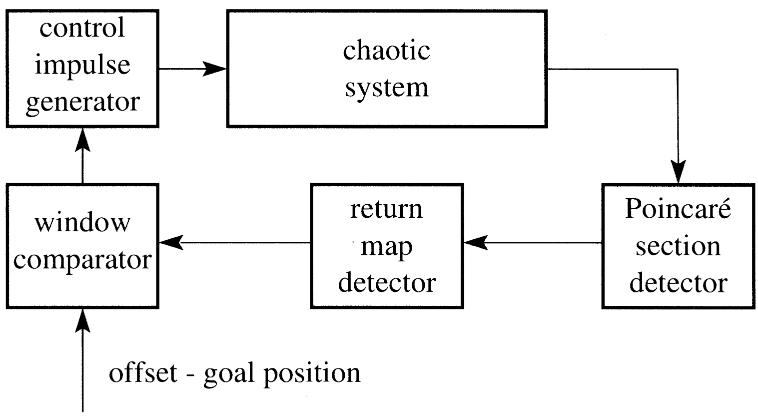

Figure 5. Block diagram of the OPF chaos controller.

the generator. Figure 4 shows the chaotic attractor and two sample orbits controlled within the chaos range.

(*c*) *Occasional proportional feedback: analogue circuitry for chaos control*

In real applications, a 'one-dimensional' version of the OGY method – the so-called occasional proportional feedback (OPF) method – has proven to be most efficient. This method has been successfully implemented in a continuous-time analogue electronic

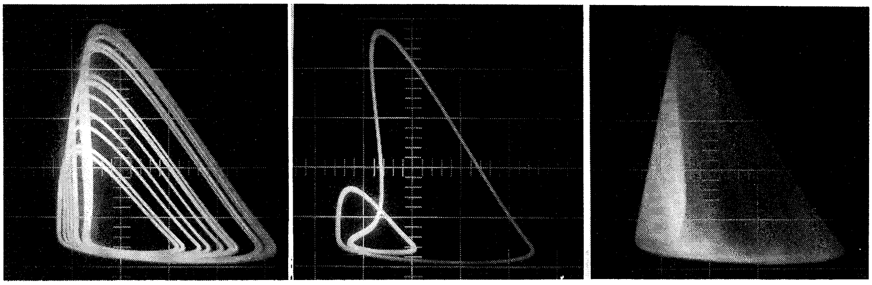

Figure 6. Chaotic attractor observed in the Colpitts oscillator and two of the controlled periodic orbits.

circuit and used in a variety of applications ranging from stabilization of chaos in laboratory circuits (Hunt 1993) to stabilization of chaotic behaviour in lasers (Corcoran 1992; Roy *et al.* 1990). The OPF method (see figure 5) is applicable in any real chaotic systems (also higher-dimensional ones) where output can be measured in an electronic way and the control signal can be applied via a single electric variable. All the signal processing is done in an analogue way thus is quick and efficient. Processing here means detecting position of a one-dimensional projection of a Poincaré section (map) which can be accomplished by window comparators and track-and-hold circuitry. Figure 6 shows results obtained in a laboratory experiment with a Colpitts oscillator operating in a chaotic mode (Kennedy 1994). The control signal has been applied via a voltage-controlled resistor. The accessible goal trajectories have to be determined by trial and error. Thus applicability of the control strategy is limited to systems in which the goal is supression of chaos without more strict requirements.

6. 'Chaos-to-chaos' control: synchronization as a control problem

It should be pointed out that synchronization and control problems of chaotic systems have common points. In particular, synchronization can be considered as a particular type of control problem in which the goal of control is to track (follow) the desired (input) chaotic trajectory. It is only very recently that such a control problem has been recognized in control engineering. The linear coupling technique and the linear feedback approach to controlling chaos can be applied for obtaining any chosen goal, no matter whether it is chaotic, periodic or constant in time. Using the approach described by (Kocarev *et al.* 1993; Kocarev & Ogorzałek 1993) we can even think of synchronizing/controlling chaotic systems to chaotic trajectories being solutions of a qualitatively different chaotic system. We believe that this kind of chaotic synchronization – control to a chaotic goal – could lead to new developments and possibly new applications.

7. Discussion

The control problems existing in the domain of chaotic systems are far from being fully identified, to say nothing about their solutions. Due to extreme richness of the phenomena, one can treat every single such problem as a new challenge for scientists and engineers. Among many problems to be solved let us mention here the basic ones: How can the methods already developed be used in real applications? What are the limitations in terms of convergence, initial conditions, etc., of these methods? What are the limitations in terms of system complexity, possibilities of implementations? Are these methods useful in biology or medicine?

New application areas open up thanks to these new developments in various aspects of controlling chaos; these include neural signal processing (Mpitsos & Burton 1992), biology and medicine (Nicolis 1987; Garfinkel *et al.* 1992; Schiff *et al.* 1994) and many others. Sensitive dependence on initial conditions – one of the key features of chaotic systems – offers yet another fantastic control possibility which is now termed as 'targetting' (Kostelich *et al.* 1993; Shinbrot *et al.* 1992); the desired point in the phase space is reached by piecing in a controlled way fragments of chaotic trajectories. This method has already been applied successfully for directing satellites to desired positions using infinitesimal amounts of fuel (see Farquhar *et al.* 1985 and references within) – chaos did the job!

Finally, we stress that almost all chaotic systems known to date have strong links with electronic circuits; variables are sensed in an electric or electronic way, identification, modelling and control is carried out using electric analogues, electronic equipment and electronic computers are used as sensors and transducers. This presents an infinite wealth of opportunities for researchers in the domain of electronics, and not only for theoreticians. For the interested reader we included a comprehensive bibliography containing also several review papers presenting the subject in a wider aspect.

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